How Modern Algebra was used in Economic Science in the 1950s:

Breaking the Glass Wall to the Scientific Acceptance

(General Equilibrium Theory (2): the Existence Question)

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This paper investigates how Japanese mathematical economists studied the questions relating to the existence of a general equilibrium and fixed point theorems (FPTs), which were keys to the proof, from the 1940s till the early 1960s. We focus on Hukukane Nikaido (1923-2001) and Hirofumi Uzawa (b.1928) and trace their direct connection with John von Neumann (1903-1957) and Kenneth J. Arrow (b.1921). Then we first reconstruct the process in which the cannon of modern neoclassical economics, namely Walrasian general equilibrium theory, was established through the use of modern algebra in the 1950s. Second, we show how the Japanese overcame the glass wall to the international community which had been established by the swift circulation of refereed economics journals like *Econometrica*. They owed much to von Neumann and active members of the Econometric Society including Arrow.
1. Japanese Economists and General Equilibrium Approach in the 1940-60s

The proof of existence, stability and uniqueness are important topics for the study of general equilibrium theory. In the 1950s, the proof of the existence of a general equilibrium utilized topology and fixed point theorems (FPTs) or set theory and the convex set method, which were mathematical tools different from those used for the proof of stability (like systems of ordinary differential equations and Liapunov theory). Seiji Takizawa (1991: 1067) gives an intuitive exposition and said, "Topology is the geometry that studies unchangeable characteristics in one-to-one bicontinuous transformations (Both mapping and reverse mapping are continuous). Roughly speaking, it is the geometry on an elastic plane. It considers if two points are connected regardless of whether the lines are straight or curved, long or short" (My translation). Moreover, the research of stability analysis was promoted by a different group of scholars prior to the study of the so-called existence question. In the 1940s several Japanese economists made important contributions to stability analysis, most of them written in Japanese but comparable to the studies which were developed in North America and Europe in the 1950s (Ikeo 1994b, 2006). As is known well, several Japanese economists made significant contributions to the study of the existence question in the 1950s. In contrast, it is less known how they embarked on this study, while making cutting edge contributions. This paper uses not only my personal communications with several scholars but also the correspondence among the economists of the day kept in the Special Collection Library of Duke University (see Weintraub, Meardon, Gayer and Banzhaf 1998) and the Yasui Library of Saitama University.

The history of the study of the existence question is so complicated that we also have to pay attention to the equally complicated history of modern algebra. Modern algebra was rapidly developed by formalist mathematicians including Emmy Nöther, a student of David Hilbert (1862-1943), from the mid 1920s on. Then it was spread by textbooks such as B.L. van der Waerden’s *Modern Algebra* (1930-31, in German) and K. Shoda’s *Abstract Algebra* (1932, in Japanese), both of which were based on Nöther’s lecture notes at Göttingen University. A number of Japanese mathematicians studied in Göttingen, Berlin and Vienna from the 1920s to the 1930s, and therefore Japanese scholars who began to
study mathematics prior to 1960 mastered the mathematics well which had been discussed and published in German (Ikeo 2006). In this respect, the Japanese studied mathematics in a tradition different from those who had studied mathematics mainly in France and North America, where the structural trend in mathematics was identified with the name of Nicolas Bourbaki, a group of mathematicians, in the 1940s and 1950s.

When we look into the conditions in which Japanese scholars became involved in the study of the existence question, we find that the swift circulation of scientific journals, most of which were refereed and published in the United States, was crucial. The Japanese scholars were not exceptions and began to work on similar subjects within a few years of the conclusion of the Asian-Pacific campaign in August 1945 as did those economists who made it a rule to read every issue of the scientific journals. In contrast, no refereed journal of economics with free submission had existed in Japan prior to 1960, when Kikan Riron Keizaigaku, later renamed Japanese Economic Review, introduced a referee system for the first time. On the other hand, each university had its own organ, called Kiyo, which was usually closed to scholars outside of the university or the department. Therefore, in the 1950s, a mathematical economist like Hukukane Nikaido, who had graduated from the department of mathematics, had no opportunity to publish his papers in any journal of economics in Japan, and had no choice but to submit his papers to scientific journals published abroad. In general, without such scientific journals by way of free submission and a referee system, the Japanese economists could not contribute their scientific works to the international forum of economists. Unfortunately there were occasionally unlucky decisions in the refereeing process such as the rejection of Nikaido's existence paper by Econometrica, which we will discuss in the penultimate section. We argue that the submission of his existence papers was not handled fairly based on the evidence remaining in the Nicholas Georgescu-Roegen Papers at Duke University.

Section 2 summarizes the current condition of the historical study of the proof of the existence of a general competitive equilibrium and its related subjects. Section 3 discusses how Japanese scholars found the intensive use of modern algebra in economic science and the progress of the study of existence question. Section 4 gives focuses on H. Nikaido's case in particular and discusses how he initiated the study of the existence question. Section
5 draws two conclusions.


In retrospect, Léon Walras developed the concept of a price system in the context of interrelated markets within the economy by utilizing a system of simultaneous equations in his *Eléments d'économie politique pure* (1874-77). Later Gustav Cassel's simplified system of general equilibrium in the fourth edition of his *Theoretische Sozialökonomie* (1927) that gave an opportunity to seriously investigate the existence of competitive equilibrium.1 Around 1930, the problems in Cassel's own handling of price determinacy became an issue in both Central Europe and Japan.2 Mathematicians and those economists who were less allergic to mathematical arguments became interested in general equilibrium approach.

The academic study of economic theories was interrupted by WWII in Europe and Japan. After the conclusion of the war, American and European scholars resumed their scientific research, cooperating with each other in the United States through the organization of conferences supported by the U.S. Government. The Dutch economist Tjalling C. Koopmans was one of the leaders in the rapid development of activity analysis and mathematical economics in the 1940s and 1950s. In the path-breaking conference volume, *Activity Analysis of Production and Distribution* (1951), Koopmans briefly summarized the discussions among European economists in the 1930s on generalizations of the Walrasian general competitive equilibrium analysis as follows:

Neisser (1932) and von Stackelberg (1933) raised questions of existence and

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1 Cassel presented his simplified Walrasian system for the first time in his 'Introduction to the theory of price' (1899, in German). The system is known as the Cassel-Walras System. I thank Henk W. Plasmeijer for bringing Cassel (1899) to my attention. There are two English editions (1923; 1932) of Cassel’s *Theoretische Sozialökonomie*, both of which have the section “Arithmetical Treatment of the Problem of Equilibrium”.

2 For example, Kei Shibata (1930, in Japanese) explained one of the formal problems in Cassel's simplified system of general equilibrium, which was pointed out three years later in H.v. Stackelberg's 'Two comments on Gustav Cassel's theory of price' (1933, in German).
uniqueness of a solution to Cassel's formulation of the Walrasian system, with reference in particular to the requirement that prices and rates of production be represented by nonnegative numbers. In a mathematical seminar conducted in Vienna by Karl Menger, Schlesinger (1935) formulated a suggestion, made also by Zeuthen (1933), that economic theory should explain not only the nonnegative prices and quantities produced of scarce goods but also which goods are scarce and which are free (i.e., have a zero price). Wald (1935, 1936a, b) proved the existence and uniqueness of a solution to an equation system expressing this problem. (Koopmans ed. 1951: 1)

This summary provided the common understanding of the development of general equilibrium theory in the 1930s. A little later, one pair of economists and three individual economists independently proved the existence of a competitive economy with the use of a particular FPT.


Brouwer's FPT for point-to-point continuous transformations (1911)


Eilenberg and Montgomery's FPT for set-to-set continuous transformations (1946)


Kakutani's FPT for point-to-set continuous transformations (1941)


Debreu (1982) clearly differentiates the two approaches to the question of the existence of a competitive equilibrium for an economy. One was the "Simultaneous Optimization Approach" taken in Arrow and Debreu (1954), in which the existence question was transformed into the question of "existence of an equilibrium for a social
system composed of a finite set of agents simultaneously seeking to maximize their utility functions, or, more generally, to optimize with respect to their preference relations" (Debreu 1982: 715). Or, this was called the "Abstract Economy Approach" (Border 1985: 95). Another was the "Excess Demand Approach" focusing on the excess demand correspondence of the economy, taken in Gale (1955) and Nikaido (1956). We will discuss the proving process in section 4.

Then Hirofumi Uzawa in his 'Walras's existence theorem and Brouwer's fixed-point theorem' (1962) proved that the two theorems in the title were equivalent. Although he was at Stanford University, the paper appeared in Kikan Riron Keizaigaku (Economic Studies Quarterly), which became a refereed journal in 1960.³ Uzawa followed the excess demand approach taken in Gale (1955) and Nikaido (1956), but restated Walras's Existence Theorem and Brouwer's Fixed-Point Theorem more simply. Uzawa's formulation went as follows.

There are $n$ commodities, $p$ is a price vector, and $x$ is a commodity bundle. Price vectors are assumed to be nonzero and nonnegative; commodity bundles are arbitrary $n$-vectors. $P$ and $X$ are the sets of all price vectors and of all commodities bundles. The excess demand function $x(p)$ is a mapping from $P$ into $X$. A price vector $\bar{p}$ is called an equilibrium if $x_i(\bar{p}) \leq 0$, with equality unless $\bar{p}_i = 0$.

**Walras's Existence Theorem.** Let an excess demand function $x(p)$ satisfy the following conditions:

(A) $x(p)$ is a continuous mapping from $P$ into $X$.

(B) $x(p)$ is homogeneous of order 0; that is, $x(tp) = x(p)$, for all $t > 0$ and $p \in P$.

(C) Walras's law holds: $\sum_{i=1}^{n} p_i x_i(p) = 0$, for all $p \in P$.

Then there exists at least an equilibrium price vector $p$ for $x(p)$.

component sums are one: \( \prod = \left\{ \pi = (\pi_1, \ldots, \pi_n); \pi \geq 0, \sum_{i=1}^{n} \pi_i = 1 \right\} \).

**Brouwer's Fixed Point Theorem.** Let \( \varphi(\pi) \) be a continuous mapping from \( \prod \) into itself. Then there is at least a fixed-point \( \pi \) in \( \prod : \pi = \varphi(\pi) \).

**Equivalence Theorem.** Walras's Existence Theorem and Brouwer's Fixed-Point Theorem are equivalent.

As Uzawa said, it had been already well established that Brouwer's FPT implies Walras's Existence Theorem. He constructed an excess demand function which satisfied conditions (A), (B), and (C). With dividing a price by the summation of prices, Uzawa neatly proved that Walras's Existence Theorem implies Brouwer's FPT (see Appendix). Before its publication, Uzawa sent a copy of his equivalence theorem paper to Kenneth Arrow at Stanford University and Arrow immediately decided to invite Uzawa to Stanford.\(^4\) In the 1950s, Nikaido and Uzawa joined Arrow's project on the Efficiency of Decision Making in Economic Systems at Stanford, which was backed by the Office of Naval Research (ONR).\(^5\) Thus Japanese economists played active roles in the study of the existence and stability of a general equilibrium in a competitive economy, two sector growth models and welfare economics although they were feeling uncomfortable with the source of research fund.

Needless to say, the study of the existence question continued as surveyed in Debreu (1982). Leading economists of the world worked individually to elaborate the

\(^4\) Uzawa's result implies that "any algorithm that is guaranteed to compute equilibria of arbitrary economies specified in terms of aggregate excess demand functions must be guaranteed to compute fixed points of arbitrary mapping of the simplex into itself" (Kehoe 1991: 2055-56). Later in the 1960s such an algorithm was developed by H.E. Scarf. Then Scarf's algorithm method was exploited in the proof of the existence of competitive equilibrium in Arrow and Hahn's advanced textbook *General Competitive Analysis* (1971).

\(^5\) Prior to the publication of Uzawa (1962), Takashi Negishi’s masterpieces (1960, 1961) became available (see Ikeo 2006, 2009; Kawamata 2009). Negishi, Ken-ichi İnada (b.1925) and Hajime Oniki also joined Arrow's project (The K.J. Arrow Papers at Duke University; Ikeo 1996).
theory, whereas many of them had studied in the U.S. The chance of studying general
equilibrium theory is open to economists of every nationality, although Nikaido had
difficulty in getting his research results published in the 1950s. In other words, nationality
does not matter for scientific study of economic theory, although the chance of publishing
research results was much more limited in Japan than the U.S.

Several historians of economic thought have been working on the intriguing
development of general equilibrium analysis. The intellectual legacy of general equilibrium
analysis from the German-speaking world has been often represented by the seminar works
of Karl Menger (1902-85), son of Carl Menger (1840-1921), as noted in Koopmans (ed.
1951). Menger's colloquium was studied in E.R. Weintraub's 'The existence of a
Menger's mathematical colloquium' (1989) and 'The school of mathematical formalism and
the Viennese circle of mathematical economists' (1991).6

The development of the study of the existence question was known and remembered
by many general equilibrium theorists of the time, who were the majority of mathematical
economists in the 1950s. The topic was first studied historically in E.R. Weintraub's 'The
existence of a competitive equilibrium: 1930-1954' (1983) and General Equilibrium
Analysis: Studies in Appraisal (1985: chapter 6), and he examined the research line of the
existence of a competitive equilibrium including the part summarized by Koopmans and
leading to K.J. Arrow, G. Debreu and L. McKenzie. Interestingly Weintraub found that
Arrow, Debreu and McKenzie proved the existence of general equilibrium independently of
Wald (1935; 1936ab). Therefore, it is not surprising that Nikaido read neither Wald's papers
nor Kazuo Midutani's 'Comments on Wald's proof of the uniqueness of the solution for the
Cassel-Schlesinger system of production' (1939, in Japanese), when he started working on
this question around 1950. Yet later, Nikaido (1968: 249) stated, "Naturally, the pioneering
work of Wald (1935, 1936a) which proved the existence of equilibrium for a Casselian
system, is remarkable". It is common among academicians to accord respect to a precedent

Yukio Mimura (a mathematician who taught Shizuo Kakutani at the Imperial University of
Osaka), and Yuzo Yamada attended K. Menger’s colloquium in the 1930s.
contribution when they find it even if they studied the subject independently of it.

B. Ingrao and G. Israel (1990) made a detailed study of Debreu's line to the existence question, including the French mathematical tradition, and regarded Nicholas Bourbaki (1939-) as the important intellectual background of Debreu. Nicholas Bourbaki was the name given to a group of French mathematicians, formed in the mid 1930s, who started to use the axiomatic method consciously in French under the influence of modern algebra intensively discussed and rapidly developed in German. This group was later joined by American mathematicians such as Samuel Eilenberg. Weintraub and P. Mirowski (1994) discussed the philosophical background of mathematical structurism. They showed that it was Debreu who introduced Bourbakism into the community of mathematical economists in the United States.7

On the other hand, until around 1939 many Japanese leading mathematicians had studied mathematics in the German-speaking world and therefore the Japanese scholars in general who began to study mathematics prior to 1960 mastered well the mathematics which had been discussed and published in German. In this respect, the Japanese studied mathematics in a tradition different from those who had studied mathematics in other areas such as France and North America. For example, Kazuo Midutani, Shizuo Kakutani and Hukukane Nikaido studied mathematics through reading literatures written in German and in contrast to Debreu none of the three was not interested in Bourbaki's 'new' mathematics. We argue that the case of the Japanese scholars expands the variety of routes to the application of a fixed point theory to the solution of the existence question in a competitive economy. This demonstrates that there was not a single path to the goal for the proof of the existence of general competitive equilibrium.

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3. Japanese Mathematical Economists and the Existence Question

Within a few years after the end of WWII, the Japanese were working on similar subjects, in both mathematics and economics, as the mathematicians and economists abroad thanks to the prompt circulation of scientific, refereed journals and important conference volumes. Around 1950 in Japan, Hiroshi Furuya (1920-1957), a student of Takuma Yasui, noticed the strong trend toward the thorough mathematization of economics. He invited mathematics students such as Tamotsu Yokoyama (b.1921), Kenichi Inada and Hiroyumi Uzawa to the community of economists on one hand, and strongly advised economics graduates to study mathematics on the other. Another mathematics student, Hukukane Nikaido, realized that John von Neumann's and Kenneth J. Arrow's economic works were different from those of J. R. Hicks and Paul Samuelson's, which were based on calculus. In the new approach, the abstract economy was modeled based on the knowledge of modern algebra (to establish the existence of general equilibrium and to clarify the welfare aspects of the competitive economy).

The first conference on mathematical programming had been held at the University of Chicago in 1949. The proceedings entitled *Activity Analysis of Production and Allocation* (Koopmans ed. 1951) were published as a Cowles Commission monograph and soon copies arrived in Japan. Their themes were, directly or indirectly, related to the best allocation of limited means toward desired ends. The organizer was T.C. Koopmans. Other contributors were Kenneth J. Arrow, Paul A. Samuelson, Robert Dorfman, Nicholas Georgescu-Roegen, Oskar Morgenstern, and Herbert A. Simon; mathematicians Albert W. Tucker, Harold W. Kuhn and David Gale; George B. Dantzig, Murray A. Geisler and Marshall K. Wood from the U.S. Department of the Air Force. Francis W. Dresch from the U.S. Naval Proving Ground, Walter H. Keen and Fred D. Rigby from the U.S. Department of the Navy also participated.

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8 This section is summarized mostly from the writings of Takuma Yasui (1909-95) rather than my personal communications with him. Yasui had studied the conditions for the stability of a competitive equilibrium with the use of a system of ordinary differential equations and Liapunov theory in Japan in the 1940s (Ikeo 1994b, 2006). But it uses my personal communications with Nikaido.
Also in 1951, K. J. Arrow's 'An extension of the basic theorems of classical welfare economics' appeared in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* edited by Jerzy Neyman. Thanks to the ONR and other institutions, the symposium had been held over a fortnight with the participation of several scholars from abroad. Arrow reviewed Pareto optimality from the viewpoint of convex set theory. Gerard Debreu in his 'The coefficient of resource utilization' (1951), independently of Arrow, embarked on the set-theoretic and convex-set method in the study of the optimality of competitive equilibrium. At first, the new approach taken by these mathematical economists seemed to refute the differential calculus basis for economics. Then, mathematically-trained scholars increasingly entered the field of mathematical economics on one hand, and theoretical economists found it necessary to study topology themselves on the other.

Those mathematical economists who had recognized the problem of existence of a competitive equilibrium were directly stimulated by John Nash's 'Non-cooperative games' (1951). Nash called the \( n \)-person games, which were developed in von Neumann and Morgenstern's *Theory of Games and Economic Behavior* (1944), cooperative. Their theory was based on an analysis of the interrelationships of various coalitions which can be formed by the players of the game. Nash embarked on the theory of non-cooperative games, which was based on the absence of coalitions or on the assumption that each participant acted independently, without collaboration or communication with any of the others (Nash 1996: 286). Nash (1951) proved the existence of equilibrium points by the use of Brouwer's FPT for point-to-point transformations, whereas he used Kakutani's theorem for point-to-set transformations in his previous paper 'Equilibrium points in n-person games' (1950). Nash (1951) constructed (or interpreted) a continuous transformation \( T \) of the space of \( n \)-tuples such that the fixed points of \( T \) are the equilibrium points of the game.

On the other hand, mathematicians were also working hard on topology, and the treatment of FPTs had been further improved (generalized) since Kakutani (1941). Samuel

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10 This paper was 'a more polished version of his doctoral thesis' (Nash 1996: 32) at
Eilenberg, an active American member of Nicolas Bourbaki, and Deane Montgomery in their 'Fixed point theorems for multi-valued transformations' (1946) extended Solomon Lefschetz's trace formula to set-valued mapping. They proved that if \( Y \) is an acyclic absolute neighborhood retract and \( f \) is an upper hemi-continuous mapping which assigns to each point \( y \) of \( Y \) an acyclic subset \( f(y) \) of \( Y \), then \( f \) has a fixed point, namely there is some \( y \) such that \( f(y) \) contains \( y \). Here, an acyclic set is one which has the same homology groups as does a set consisting of just one point. Eilenberg and Montgomery's FPT is the most general and included Kakutani's. Then Edward G. Begle (1950) gave another proof to the most general FPT.

Hukukane Nikaido was following this trend and knew very well what was happening in the forefront of topology. Nikaido (1959) examined the generalization of FPT in the study of systems of inequalities originated with von Neumann's works on his minimax theorem, reformulated by Kakutani and developed by Eilenberg, Montgomery and Begle. Nikaido (1959: 354-5) states as follows,

[S]ince . . . von Neumann's initial work attention had mainly been confined to some game problems or their variants, and no attack had ever made against relevant conjectures [the existence of a general equilibrium solution] in the orthodox mathematical economics until in recent times Arrow-Debreu [1954], McKenzie [1954], Gale [1955] and this writer [Nikaido 1956] independently and almost simultaneously gave reformulations and proofs to the most basic conjecture in the theory of general equilibrium as founded by L. Walras around the end of the last century [the 19th century]. . . . [A]s in the theory of games fixed point theorems or their equivalent propositions proved to be very helpful. It is interesting as well as significant that the minimax problems and those of economic equilibrium have some intersection in common and reveal a certain similarity between them.

Takuma Yasui participated in the Chicago meeting of the Econometric Society held on 27-29 December 1952. He was sent there by the Science Council of Japan (Nihon Princeton University.)
Gakujutsu Kaigi) and presented his 'Nonlinear self-excited oscillations and business cycles' in the session "Macro-dynamic Models of Economic Fluctuations" on the 27th.\textsuperscript{11} On the same day he attended the session "The Theory of Games", in which K.J. Arrow and G. Debreu presented their 'Existence of an equilibrium for a competitive economy', which was discussed by L.J. Savage. On the 29th he attended the session of "Selected Papers", in which L.W. McKenzie presented his 'The existence and uniqueness of equilibrium in Graham's model of international trade'. At this meeting, Yasui for the first time learned FPTs (Yasui 1971, in Japanese: 286). Yet Yasui did not report the heated argument between McKenzie, and Arrow and Debreu on their formulations of an abstract economy and the priority of the proof (See Weintraub and Gayer 2001; Weintraub 2002). A summary of McKenzie's presentation appeared in \textit{Econometrica} of 1953, while Arrow and Debreu's was not available (Anon. 1953). Their full papers were both published in \textit{Econometrica} in 1954, with the title of McKenzie's paper changed to 'On equilibrium in Graham's model of world trade and other competitive systems'.

McKenzie proved the existence and uniqueness of competitive equilibrium in Frank D. Graham's model for world trade by using Kakutani's FPT. The production aspect of the model was represented by a linear activity model in which the primary goods are the labor supplies of the several countries. McKenzie frequently emphasized that the method of his proof was sufficiently general that the restrictive assumptions in Graham's model could be replaced by less restrictive ones. Thus, his results might be applied to other models of competitive economy. His proof of existence of an equilibrium point was given by resorting to the knowledge of topology for the first time. This proof was supplemented by the mathematical appendix. McKenzie did not refer to John Nash's papers on game theory, which were very important for other economists in solving the existence question.

Arrow and Debreu in the published version used set-theoretical techniques to specify the precise assumptions of a competitive economy as the basic starting point. They confined themselves to proving the existence of competitive equilibrium and extended

\textsuperscript{11} Robert H. Strotz, the organizer, and Martin Brofenbrenner helped Yasui participate in the Chicago meeting of and present his paper. Their correspondence remains in the Yasui Library of Saitama University (Ikeo 2006).
Nash's notion of an equilibrium point for a non-cooperative game to their abstract economy, which was first discussed in Debreu (1951). They discussed the question of the existence of a competitive equilibrium by constructing an abstract economy through a generalization of the concept of a game. They appealed indirectly to Eilenberg and Montgomery's FPT, although they did not refer to Eilenberg and Montgomery (1946) or Begle (1950) (Arrow's letter to Georgescu-Roegen of 12 January 1955, quoted in section 4; Nikaido 1959). Yet later Debreu's *Theory of Value* (1959: 27) referred to these mathematical papers.

Around 1954, Hukukane Nikaido in Tokyo and David Gale in Copenhagen were working on the existence question along a similar line, though independently of each other. Gale's 'The law of supply and demand' appeared in *Mathematica Scandinavica* of 1955. Gale obtained a simpler proof of the existence of an equilibrium than Arrow and Debreu (1954) by using a lemma of combinatorial topology and Kakutani's FPT. Nikaido's 'On the classical multilateral exchange problem' was published in *Metroeconomica* of 1956. In contrast to Gale, Nikaido elaborated the existence question independently of Arrow and Debreu (see the next section). A footnote in Nikaido's paper stated, "The result of this paper has been obtained independently of the important work carried out by Professors Arrow and Debreu ... and prior to its appearance in *Econometrica*, although it should be expressly acknowledged that there is much intersection" (Nikaido 1956: 135). Nikaido formulated the basic propositions of the existence of general equilibrium as a theorem relating to the excess demand correspondence in the case of multilateral exchange of many commodities. Nikaido resorted to slightly more restricted assumptions than Arrow and Debreu (1954) such as an upper hemi-continuous correspondence (which is included as an assumption to prove Eilenberg and Montgomery's FPT). Nikaido adapted the basic mapping formula so as to apply it to a model of world trade as well as to Graham's model treated in McKenzie (1954), and proved the existence of a general equilibrium solution with the direct use of Kakutani's FPT. Then Hirofumi Uzawa (1962) proved that Walras's existence theorem and Brouwer's FPT were equivalent as discussed in section 2.
4. Hukukane Nikaido -- Kenneth J. Arrow

Hukukane Nikaido's experience of the 1950s is very intriguing and shows how Japanese scholars entered the field of mathematical economics and struggled to publish their scientific papers at a time, when Japan had just reopened the academic channel to the international forum of economists after the Second World War. Yet, the reason why Nikaido became interested in mathematical economics and then the existence question was somewhat accidental.12

Nikaido was an undergraduate student in mathematics at the University of Tokyo, when he was allowed to attend Shokichi Iyanaga's seminar for graduate students. In 1948 Tsuneyoshi Seki (b.1924) began to attend Iyanaga's seminar to become a mathematical economist after he graduated from the economics department of Hitotsubashi University. Seki was interested in the question of the existence of general equilibrium which was discussed not only in M. Watanabe and M. Hisatake's Application of Mathematics to Economics (1933, in Japanese) but also in K. Menger's Ergebnisse eines mathematische Kolloquiums (Seki 1986: 334). Seki delivered a talk on von Neumann's 'Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes' (1937), which stimulated Nikaido to read the original paper in Ergebnisse (its copy was presumably owned by Ichiro Nakayama), and von Neumann and Morgenstern's Theory of Games and Economic Theory (1944) in a pirated edition. Nikaido recalled,

12 The study resulting in this section is based on both the Goergescu-Roegen Papers at Duke University and my personal communications with H. Nikaido. I took Nikaido's course on mathematical economics when I was an undergraduate student (of social studies) at Hitotsubashi University in the 1970s. He did not remember me when I began to study the history of mathematical economics resulting in this paper. Nonetheless, he was so nice that he answered almost all questions I asked. He even began to help me interpret the historical materials I showed to him. The completion of the study for this paper was delayed whenever I located new facts, some of which were contradictory to the previous reconstructions including Nikaido's. Moreover, I realized that it was necessary for me to read between lines and to conjecture the relevant things that Nikaido had decided not to tell me from the things that he did tell me.
Game theory was a new field at the time. Fixed point theorems were always used in proving existence in game theory, but it was not true of economics. I wondered why. I started to examine Hicks' *Value and Capital* (1939). It was also a pirated edition. An idea came to me from Nash's paper. Interestingly, the mathematical structure of a competitive economy is the same as that of game theory. [Nikaido, personal communication, my translation]

Nikaido first wrote papers in the mathematical line. He began with P. Alexandroff and H. Hopf's *Topologie I* (1935) and published his first note on FPTs in German (Nikaido 1954a). He also wrote two papers in the line of von Neumann's game theory and general equilibrium theory in 1952. However, he did not know what to do with his papers because it seemed him that he did not have any opportunity to get his papers published in the mostly closed economics journals of Japanese universities. As mentioned, there were no refereed economics journals with free submission in Japan until 1960. Nikaido sent the papers to von Neumann and Kakutani at Princeton University, and soon received a reply from von Neumann. Following von Neumann's comments and advice, Nikaido submitted the papers to two different journals. They were published as 'On von Neumann's minimax theorem' (1954b) in *Pacific Journal of Mathematics* and 'Note on the general economic equilibrium for nonlinear production functions' (1954c) in *Econometrica*. Nikaido kept von Neumann's letters as his treasure.13

Nikaido was in Japan working out the existence question of competitive equilibrium along with the minimax theorem in game theory, the theorem of Nash's equilibrium in non-cooperative games, the von Neumann growth model, and FPTs given by Brouwer and Kakutani. Nikaido did not know McKenzie or Arrow and Debreu's presentations on the same question at the 1952 Chicago meeting of the Econometric Society. In Japan, in June or July of 1954, when he came across McKenzie (1954) in the April issue of *Econometrica* (which would have been shipped by surface mail), Nikaido immediately submitted his first

13 However, Nikaido did not make it a custom to keep the other letters he received in the 1950s. He did not keep a copy of his own letter, either (Personal communication with Nikaido).
existence paper to *Econometrica*. Although no copy of the paper is available, we can conjecture that the title of the paper was 'Exchange equilibrium and a fixed point theorem'. This is because the record of the Japanese Econometric Society (1956) tells us that Nikaido presented a paper entitled 'Kokan-kinko to fudoten-teiri' at the annual meeting at Osaka University on 29 October 1954. The title means 'Exchange equilibrium and a fixed point theorem'. Then Nikaido found Arrow and Debreu's 'Existence of an equilibrium for a competitive economy' in the July issue of *Econometrica*.

The correspondence relating to the existence question, part of which remains in the Georgescu-Roegen papers at Duke University and Nikaido's home, tells us the treatment of Nikaido's submission to *Econometrica*. The editorial board of *Econometrica* made a typed copy of those letters it received, and made a few carbon copies of the letters it mailed.

Nikaido's first submission was rejected although his mathematical argument seemed to have high quality. Nikaido received the rejection letter of 1 October 1954 (sent by airmail) from Robert H. Strotz, the managing editor for *Econometrica*. Nikaido accepted the reasons why his paper was rejected and replied in his letter of October 7, a copy of which remains at Duke University, as follows:

> I received yesterday your letter of October 1 . . . I think the referee's comment on my manuscript judges appropriately the value of my result and I therefore understand completely your processing of my paper based on this comment. Thus I only hope that I might be able to submit a paper of more economic merit in a future opportunity.

> As to Professor McKenzie's article you mentioned I have read it, and thus my manuscript was written with the reference to it, while unfortunately I had no opportunity to read Arrow-Debreu article before having submitted the manuscript to you.

---

At the same time, Nikaido was asked to make comments on a draft of a 'Letter to the Editor' submitted by Cecil Phipps, the mathematician who was one of the two referees for Arrow and Debreu's existence paper submitted to *Econometrica*. Although Phipps was unsatisfied with the mathematical arguments made by these mathematical economists, he failed to convince the reviewers including Nikaido. The letter never appeared in *Econometrica* (Weintraub and Gayer 2001).\(^{15}\)

Nikaido submitted his second existence paper entitled 'On the classical multilateral exchange problem' to the *Econometrica* in December 1954. Strotz wrote to Nicholas Georgescu-Roegen, one of the assistant editors, on December 24 and asked him to handle the refereeing of the manuscript (Strotz's letter to Georgescu-Roegen of 24 December 1954). In turn Georgescu-Roegen chose Arrow as a referee. However, Arrow promptly wrote back to Georgescu-Roegen and the referee process came to a stop. Arrow's letter to Georgescu-Roegen of 12 January 1955 said as follows (full quotation of the text),

I have just read carefully the paper of Mr. Nikaido. Although it is an excellently written paper, I cannot recommend its publication because of its extremely close overlap with the paper that Debreu and I have published. The technique of proof is almost identical. Such simplifications as exist are due to his having made stronger hypotheses. It is true that he appeals directly to Kakutani's theorem rather than as we do indirectly to the more general Eilenberg-Montgomery theorem. However, as we note explicitly, it would be quite easy to modify our proof to make use of the Kakutani theorem and we only made use of Debreu's because it is already available in the literature.

As for more detailed I know there will be very little to make since the organization and exposition of the paper are admirable. I have not read every line in detail and there will be minor suggestions, but I do not think it worthwhile going into unless

\(^{15}\) I thank Ted Gayer for providing me with the information about the referring process of Arrow and Debreu's existence paper. Weintraub and Gayer (2001) discussed how the existence of a general competitive equilibrium was proved and established in the 1950s, and why Arrow and Debreu (1954) appeared in *Econometrica* in spite of the negative comments from the mathematician Cecil Phips, one of the two referees.
you decide to publish the paper anyway. I will, therefore, hold the manuscript for another week and, if you wish me to, I will be glad to referee it in detail. If not, I will return the manuscript to you. Perhaps it would be better to have some person other than myself or Debreu review the question of publication since it is possible that I am prejudiced. However, in all frankness, I feel quite sure of my position.

Georgescu-Roegen arrived at the same opinion as Arrow without consulting another referee and said in his letter to Arrow of 17 January 1955 as follows,

Thank you very much indeed for your prompt comments on Nikaido's paper. / After a superficial reading, I arrived exactly at the same opinion as yours, and I am glad to have it now supported by someone else. / I thought that if Nikaido believes that he brings out some additional result, not included in your paper, he might submit it as a note of a length proportionate to his contribution and without re-proving your own results. No matter what one can think about the merits of Nikaido's proof, I feel that *Econometrica* cannot afford to devote space to mere analytical refinements. / Before making this recommendation to the editor, I would like to know whether you agree with it.

While getting Arrow's agreement (in Arrow's letter to Georgescu-Roegen of January 21), Georgescu-Roegen said in his letter to Strotz of February 4 as follows,

Nikaido's proof is somewhat neater and simpler than that of Arrow-Debreu, but I feel that this merit alone does not justify its publication. It would be a very poor allocation of our resources. Indeed, his paper brings nothing new. / I understand the reason may not be well received by Nikaido and that might feel particularly dissatisfied after he sees a paper dealing only with a new proof of Arrow-Debreu results by McKenzie published in the forthcoming proceedings of the last conference on Linear programing. Notwithstanding, I do not see what we can do about it.
Nikaido was not aware of the strange refereeing process. Moreover, he barely remembered that he did not receive a rejection letter this time. Instead, he unexpectedly received a letter from Arrow and was advised to resubmit his paper to *Metroeconomica*, a journal which he had never heard of. Fortunately Nikaido's 'On the classical multilateral exchange problem' was published in *Metroeconomica* in 1956. A footnote says, "The result of this paper has been obtained independently of the important work carried on by Professors Arrow and Debreu [1954] and prior to its appearance in *Econometrica*, although it should be expressly acknowledged that there is much intersection."

Nikaido and Arrow continued to correspond. At the time Nikaido was eager to leave Japan for a better place to study, and asked Arrow if there was a possibility of his staying in the United States. Arrow's letter to Nikaido of 1 March 1955 remains at Nikaido's house. Arrow wrote as follows,

Thank you for your letter of February 21. I have been following your work with great interest and I am very impressed with its quality. I would be very happy to see you enter the field of economics and I would like to do everything in my power to help you.

Unfortunately, however, my powers are limited in this regard. I can offer you the position of research associate in a group working here under my direction for the coming year but the salary is only $2400. I believe it is possible for you to supplement this by a Fulbright Grant for travel expenses. If this arrangement appeals to you, I would feel greatly privileged to have you join us. Please let me know whether you can come beginning this coming September.

Nikaido took a chance and was appointed as Research Associate in the Applied Mathematics and Statistics Laboratory at Stanford University. According to D. Whitaker's letter to Nikaido of 15 April 1955, Nikaido's salary was 275 dollars a month for the period from October 1, 1955 to June 30, 1956. Nikaido arrived at Stanford in the summer of 1955. He was informed by Arrow that results in the same line had been achieved by David Gale
Later Debreu in his *Theory of Value* (1959: 88) noted that Nikaido independently proved the existence of a competitive equilibrium in his 1956 paper. As mentioned in section 2, Debreu differentiates Nikaido and Gale's approach, calling it the "Excess Demand Approach", from Arrow and Debreu's "Simultaneous Optimization Approach" (Debreu 1982; see also Debreu 1987: 217-8).

Then Arrow in his entry 'Economic Equilibrium' (1968) for the *International Encyclopedia of the Social Sciences* stated as follows (pp. 379-380),

Von Neumann deduced his saddle-point theorem from a generalization of Brouwer's fixed point theorem, a famous proposition in the branch of mathematics known as topology. A simplified version of von Neumann's theorem was presented a few years later by the mathematician Shizuo Kakutani, and Kakutani's theorem has been the basic tool in virtually all subsequent work. With this foundation, and the influence of the rapid development of linear programming on both the mathematical--again closely related to saddle-point theorems--and economic sides (the work of George B. Dantzig, Albert W. Tucker, Harold W. Kuhn, Tjalling C. Koopmans, and others, collected for the most part in an influential volume [Cowles Commission ... 1951]) and the work of John Nash, Jr. (1950), it was perceived independently by a number of scholars that existence theorems of greater simplicity and generality than Wald's were possible. The first papers were those of McKenzie (1954) and Arrow and Debreu (1954). Subsequent developments were due to Hukukane Nikaido and Hirofumi Uzawa, Debreu, and McKenzie.

Arrow (1968) clearly stated Nikaido's contribution to the study of the existence question, although it did not include Nikaido (1956) in the references. The material of Arrow (1968) was incorporated into chapter 1 of Arrow and Hahn's *General Competitive Analysis* (1971: 11), with reference to the particular paper, Nikaido (1956). It is worth quoting from Arrow's Foreword to Shepherd's edited book *Rejected: Leading Economists Ponder the Publication Process* (1995), 'But to suggest that the normal process of scholarship work
well on the whole and in the long run is in no way contradictory to the view that the processes of selection and sifting which are essential to the scholarly process are filled with error and sometimes prejudice. George Shepherd has seized on one aspect of the process, publication, and it is a key one in the allocation process by which the existing structure of scholarship controls new entry' (p.vii). 16

Nikaido showed me the letters he had received in the 1950s and pleasantly told me details about why and how he had come to join Arrow's project at Stanford. However, he did not talk much about the project as a whole and how he had spent his research time at Stanford. I assume there are two reasons. One reason was that he must feel uncomfortable by the fact that Arrow's project had been financially supported by ONR, which was part of the Navy. Yet he felt relieved when he learned that ONR had been essentially acting as the office of national research from 1945 until around 1957, the year in which the former Soviet Union launched Sputnik, the first unmanned space satellite and it had managed to hobble the newly established National Science Foundation (NSF) by sending Navy-related people to the top of NSF (Sapolsky 1990: 38, 54) by reading the Japanese version (1999) of Ikeo (ed. 2000). Nonetheless, it seemed me that Nikaido was hiding something which he was very reluctant to tell me but he suggested me something strange in the referring process of his existence paper (Nikaido 1956) in *Econometrica*.

5. Conclusions

Let us draw two conclusions from the historical study we have presented in this paper.

First, until the early 1950s Japanese scholars took a separate course from Arrow, Debreu and Mackenzie to the study of the so-called existence question. However, through the 1950s, mathematical economists including those in Japan took a similar procedure for proving the existence of equilibrium in a competitive economy by borrowing tools from topology and game theory. They clarified the mathematical structure of a competitive economy and the appropriate conditions which were required to claim the existence of equilibrium in a competitive economy. It was necessary to construct an abstract economy or

---

16 This passage was also quoted in Weintraub (2002).
excess demand function by using knowledge of topology including closed sets, convexity, compactness and boundedness in order to maintain that the system had a meaningful solution and to discuss the welfare aspect. As shown by Uzawa's equivalence theorem (Uzawa 1962), they were giving economic interpretations to FPTs, which were being studied by mathematicians around the same time. Thus, the cannon of modern neoclassical economics, namely Walrasian general equilibrium theory based on modern algebra, was established.

Second, Hukukane Nikaido worked out the proof of the existence of a competitive equilibrium independently of McKenzie (1954) and Arrow and Debreu (1954), both of which appeared in *Econometrica*, the most and foremost influential journal of economic science in the mid 20th century. Nikaido (1956) was published in *Metroeconomica*, after Gale (1955) came out in *Mathematica Scandinavia*. The remaining evidence tells us that Nikaido's submission of his existence paper to *Econometrica* was rejected twice but accepted by a refereed journal a few months after a paper of a similar line was published. In other words, Nikaido's existence paper was unfairly treated at *Econometrica* in the sense that a normal referring procedure did not take place. Nikaido's existence paper might have been accepted at *Econometrica* after a normal refereeing process, if the Arrow of 1968 or Debreu had refereed it. Yet thanks to Arrow, Nikaido (and other Japanese economists) obtained a better research environment in the US than in Japan.
Mathematical Appendix

Hirofumi Uzawa in his 'Walras's existence theorem and Brouwer's fixed-point theorem' (1962) proved that the two theorems in the title were equivalent. It had been already well established that Brouwer's FPT implies Walras's Existence Theorem. Uzawa (1962) proved that Walras's Existence Theorem implies Brouwer's FPT. He constructed an excess demand function \( x(p) = [x_1(p), \ldots, x_n(p)] \) by

\[
(1) \quad x_i(p) = \varphi_i \left( \frac{p}{\lambda(p)} \right) - p^*_i \mu(p), \quad (i = 1, \ldots, n, p \in P) 
\]

where

\[
\lambda(p) = \sum_{i=1}^{n} p_i 
\]

\[
\mu(p) = \frac{\sum_{i=1}^{n} p_i \varphi_i \left[ \frac{p}{\lambda(p)} \right]}{\sum_{i=1}^{n} p_i^2} 
\]

Uzawa notes that both \( \varphi_i \left( p/\lambda(p) \right) \) and \( p_i \mu(p) \) are positively homogeneous of order 0. Therefore, the excess demand function thus defined satisfies conditions (A), (B), and (C). From Walras's theorem, there is an equilibrium price \( \bar{p} \). From (1), we have

\[
(2) \quad \varphi_i \left( \frac{\bar{p}}{\lambda(\bar{p})} \right) \leq \bar{p}_i \mu(\bar{p}), \quad (i = 1, \ldots, n) 
\]

with equality unless \( \bar{p}_i = 0 \). Uzawa defines \( \bar{\pi} \) and \( \beta \) as follows,
\[
\pi = \frac{\bar{p}}{\lambda (\bar{p})}, \quad \beta = \lambda (\bar{p}) \mu (\bar{p}).
\]

Then the relation (2) is rewritten as follows,

(3) \quad \varphi_i (\bar{\pi}) \leq \beta \bar{\pi},

with equality unless \( \bar{\pi}_i = 0 \).

By summing (3) over \( i = 1, \ldots, n \), and considering that \( \bar{\pi}, \varphi_i (\bar{\pi}) \in \Pi \), we have \( \beta = 1 \); therefore,

(4) \quad \varphi_i (\bar{\pi}) \leq \bar{\pi}_i ,

with equality unless \( \bar{\pi}_i = 0 \).

The relation (4), again together with \( \bar{\pi}, \varphi_i (\bar{\pi}) \in \Pi \), implies that

\[
\varphi_i (\bar{\pi}) = \bar{\pi}_i, \quad (i = 1, \ldots, n).
\]

This means that \( \bar{\pi} \) is a fixed-point for the mapping \( \varphi_i (\bar{\pi}) \). Thus the Walras's Existence Theorem implies Brouwer's FPT. Q.E.D.
Notes

Variations of this chapter have been presented several times. After each presentation I came across other new material to be included in the conference paper. In the meantime, many scholars patiently gave me information relating to the study presented in this paper. They are Kenneth J. Arrow, Masao Fukuoka, Ted Gayer, late Shizuo Kakutani, Manabu Toda, Takashi Negishi, E. Roy Weintraub, late Hukukane Nikaido and late Takuma Yasui. Koichi Hamada helped me contact Kakutani. Paul Pecorino gave me comments on the manuscript. Fragments of this paper were given at the microeconomics workshop at the University of Tokyo in May 1994, at the annual meeting of the History of Economics Society in Babson College, Boston, at the economics workshop in Tokyo Keizai University in June 1994, at the tenth World Congress of the International Economic Association in Tunis in December 1995, at the Third European Conference on the History of Economics in Athens in April 1997, and at the Duke Workshop on the history of political economy in September 1997. Jan van Daal, Mary Ann Dimand, Takashi Negishi, Robin Neill, Christian Schmidt, Nancy Wulwick, Henk W. Plasmeijer, Akira Yamazaki and other participants gave me good questions and related information. I thank all of them. Needless to say, the remaining errors are my own.

Personal Communications:
Masao Fukuoka at Keio University in Tokyo on February 1, 1993.
Shizuo Kakutani at Yale University in New Haven on January 5 and April 3-4, 1995.
Takuma Yasui at Kwansei Gakuin University in Kobe on October 13, 1990.
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