A SECOND COURSE IN STATISTICS
REGRESSION ANALYSIS

Eighth Edition

WILLIAM MENDENHALL
University of Florida

TERRY SINCICH
University of South Florida

Pearson
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Online

SAS Tutorial

SPSS Tutorial

MINITAB Tutorial

R Tutorial
Preface

Overview

This text is designed for two types of statistics courses. The early chapters, combined with a selection of the case studies, are designed for use in the second half of a two-semester (two-quarter) introductory statistics sequence for undergraduates with statistics or non-statistics majors. Or, the text can be used for a course in applied regression analysis for masters or Ph.D. students in other fields.

At first glance, these two uses for the text may seem inconsistent. How could a text be appropriate for both undergraduate and graduate students? The answer lies in the content. In contrast to a course in statistical theory, the level of mathematical knowledge required for an applied regression analysis course is minimal. Consequently, the difficulty encountered in learning the mechanics is much the same for both undergraduate and graduate students. The challenge is in the application – diagnosing practical problems, deciding on the appropriate linear model for a given situation, and knowing which inferential technique will answer the researcher’s practical question. This takes experience, and it explains why a student with a non-statistics major can take an undergraduate course in applied regression analysis and still benefit from covering the same ground in a graduate course.

Introductory Statistics Course

It is difficult to identify the amount of material that should be included in the second semester of a two-semester sequence in introductory statistics. Optionally, a few lectures should be devoted to Chapter 1 (A Review of Basic Concepts) to make certain that all students possess a common background knowledge of the basic concepts covered in a first-semester (first-quarter) course. Chapter 2 (Introduction to Regression Analysis), Chapter 3 (Simple Linear Regression), Chapter 4 (Multiple Regression Models), Chapter 5 (Model Building), Chapter 6 (Variable Screening Methods), Chapter 7 (Some Regression Pitfalls), and Chapter 8 (Residual Analysis) provide the core for an applied regression analysis course. These chapters could be supplemented by the addition of Chapter 10 (Time Series Modeling and Forecasting), Chapter 11 (Principles of Experimental Design), and Chapter 12 (The Analysis of Variance for Designed Experiments).

Applied Regression for Graduates

In our opinion, the quality of an applied graduate course is not measured by the number of topics covered or the amount of material memorized by the students. The measure is how well they can apply the techniques covered in the course to the solution of real problems encountered in their field of study. Consequently, we advocate moving on to new topics only after the students have demonstrated ability (through testing) to apply the techniques under discussion. In-class consulting sessions, where a case study is presented and the students have the opportunity to diagnose the problem and recommend an appropriate method of analysis, are very
helpful in teaching applied regression analysis. This approach is particularly useful in helping students master the difficult topic of model selection and model building (Chapters 4-8) and relating questions about the model to real-world questions. The seven case studies (which follow relevant chapters) illustrate the type of material that might be useful for this purpose.

A course in applied regression analysis for graduate students would start in the same manner as the undergraduate course, but would move more rapidly over the review material and would more than likely be supplemented by Appendix A (Derivation of the Least Squares Estimates), Appendix B (The Mechanics of a Multiple Regression Analysis), and/or Appendix C (A Procedure for Inverting a Matrix), one of the statistical software Windows tutorials available at the course website (SAS, SPSS, MINITAB, or R), Chapter 9 (Special Topics in Regression), and other chapters selected by the instructor. As in the undergraduate course, we recommend the use of case studies and in-class consulting sessions to help students develop an ability to formulate appropriate statistical models and to interpret the results of their analyses.

Features

1. **Readability.** We have purposely tried to make this a teaching (rather than a reference) text. Concepts are explained in a logical intuitive manner using worked examples.

2. **Emphasis on model building.** The formulation of an appropriate statistical model is fundamental to any regression analysis. This topic is treated in Chapters 4-8 and is emphasized throughout the text.

3. **Emphasis on developing regression skills.** In addition to teaching the basic concepts and methodology of regression analysis, this text stresses its use, as a tool, in solving applied problems. Consequently, a major objective of the text is to develop a skill in applying regression analysis to appropriate real-life situations.

4. **Real data-based examples and exercises.** The text contains many worked examples that illustrate important aspects of model construction, data analysis, and the interpretation of results. Nearly every exercise is based on data and research extracted from a news article, magazine, or journal. Exercises are located at the ends of key sections and at the ends of chapters.

5. **Case studies.** The text contains seven case studies, each of which addresses a real-life research problem. The student can see how regression analysis was used to answer the practical questions posed by the problem, proceeding with the formulation of appropriate statistical models to the analysis and interpretation of sample data.

6. **Data sets.** The online resource provides complete data sets that are associated with the case studies, exercises and examples. These can be used by instructors and students to practice model-building and data analyses.

7. **Extensive use of statistical software.** Tutorials on how to use any of four popular statistical software packages – SAS, SPSS, MINITAB, and R – are provided online. Printouts associated with the respective software packages are presented and discussed throughout the text.

8. **End-of-Chapter Summaries.** Important points are reinforced through flow graphs (which aid in selecting the appropriate statistical method) and boxed notes with key words, formulas, definitions, lists, and key concepts.
New to the 8th Edition

Although the scope and coverage remain the same, the eighth edition contains several substantial changes, additions, and enhancements:

1. **New and Updated Case Studies.** *Case Study 2: Modeling Sale Prices of Residential Properties,* has been updated with current data. A new case study (*Case Study 7: Voice Versus Face Recognition – Does One Follow the Other?*) now follows the chapter on analysis of variance.

2. **Real Data-based Exercises.** Many new and updated exercises, based on contemporary studies and real data in a variety of fields, have been added. Most of these exercises foster and promote critical thinking skills.

3. **Statistical Software Output.** All statistical software printouts shown in the text have been updated to reflect the most recent version of the software: Minitab, SAS, and SPSS.

4. **Updated Statistical Software Tutorials.** They can be found at the following website: www.pearson.com/math-stats-resources. The text’s online resource provides updated instructions on how to use the Windows versions of SAS, SPSS, MINITAB, and R. Step-by-step instructions and screen shots for each method presented in the text are shown.

5. **Updated and New Sections in Chapter 9: Special Topics in Regression.** The section on logistic regression (Section 9.6) has been expanded. A new section (Section 9.7) on Poisson regression has been added. And, in addition to ridge regression, Section 9.8 now includes a discussion of Lasso regression.

Numerous less obvious changes in details have been made throughout the text in response to suggestions by current users of the earlier editions.

Supplements

The text is also accompanied by the following supplementary material:

1. **Instructor’s solutions manual.** The instructor’s exercise solutions manual presents the full solutions to the other half (the even) exercises contained in the text. For adopters, the manual is complimentary from the publisher.

2. **Data Files.** They can be found at the book’s resource website: www.pearson.com/math-stats-resources. The text’s online resource provides data files for all data sets marked with a data (珥) icon in the text. These include data sets for text examples, exercises, and case studies. The data files are saved in “.csv” format for easy importing into statistical software such as R, as well as in SAS (“.sas7bdat”), SPSS (“.sav”), and Minitab (“.mtw”) format.

Acknowledgments

We want to thank the many people who contributed time, advice, and other assistance to this project. We owe particular thanks to the many reviewers who provided suggestions and recommendations at the onset of the project and for the succeeding editions (including the 8th):
Jack Miller (University of Michigan)
Scott Grimshaw (Brigham Young University)

Liam O’Brien (Colby College)
Subarna K Samanta (The College of New Jersey)
Wolde Woubneh (Kean University)
Alan Huebner (University of Notre Dame)
Jen-Wen Lin (University of Toronto)
Karen Keating (Kansas State University)
Seamus Freyne (Mississippi State University)
Martin Tanner (Northwestern University)
Rebecca L. Pierce (Ball State University)
Julius Esunge (University of Mary Washington)
Brant Deppa (Winona State University)
Ross Hosky (Appalachian State University)
David Holmes (College of New Jersey)
Patrick McKnight (George Mason University)
David Kidd (George Mason University)
W.R. Stephenson (Iowa State University)
Lingyun Ma (University of Georgia)
Pinyuen Chen (Syracuse University)
Gokarna Aryal (Purdue University, Calumet)
Monnie McGee (Southern Methodist University)
Ruben Zamar (University of British Columbia)
Tom O’Gorman (Northern Illinois University)
William Bridges, Jr. (Clemson University)
Paul Maiste (Johns Hopkins University)
Mohammed Askalani, Mankato State University (Minnesota)
Ken Boehm, Pacific Telesis (California)
Andrew C. Brod, University of North Carolina at Greensboro
James Daly, California State Polytechnic Institute at San Luis Obispo
Assane Djeto, University of Nevada - Las Vegas
Robert Elrod, Georgia State University
James Ford, University of Delaware
Carol Ghomi, University of Houston
James Holstein, University of Missouri at Columbia
Steve Hora, Texas Technological University
K. G. Janardan, Eastern Michigan University
Thomas Johnson, North Carolina State University
Ann Kittler, Ryerson College (Toronto)
James T. McClave, University of Florida
John Monahan, North Carolina State University
Kris Moore, Baylor University
Farrokh Nasri, Hofstra University
Robert Pavur, University of North Texas
P. V. Rao, University of Florida
Tom Rothrock, Info Tech, Inc.
Ray Twery, University of North Carolina at Charlotte
Joseph Van Matre, University of Alabama at Birmingham
William Weida, United States Air Force Academy
Dean Wichern, Texas A&M University
James Willis, Louisiana State University

We are particularly grateful to Charles Bond, Evan Anderson, Jim McClave, Herman Kelting, Rob Turner, P.J. Taylor and Mike Jacob, who provided data sets and/or background information used in the case studies.